SYNTHESIS OF GENERAL TOPOLOGY MULTIPLE COUPLED RESONATOR FILTERS BY OPTIMIZATION

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ABSTRACT

A synthesis procedure, using optimization, for multiple coupled resonator filters having general topology and general response is described. The error function for the optimization is based on the values of the characteristic function at its zeros and poles. The optimization is performed directly on the element values of the coupling matrix. Convergence of the optimization is extremely fast and nearly independent of the starting coupling matrix. Examples of the design of practical filters with symmetric or asymmetric responses and topology are presented.

INTRODUCTION

Synthesis procedures for multiple coupled resonator filters of general response have previously been developed [1-8]. These procedures generally belong to one of two categories. The first category is the classical synthesis procedure [1-5], in which a synthesis cycle is established to extract an elementary section from the filter's transfer function, leaving a lower degree realizable transfer function. The same synthesis cycle is then repeated until the degree of the remainder transfer function becomes zero. The resulting coupling matrix from the classical synthesis has a given topology that corresponds to the generalized cascade of the extracted sections. The second category [6-9] creates, from the filter's transfer function, a coupling matrix with arbitrary topology. Successive planer (2-dimensional) similarity transformations are then applied to the coupling matrix to reduce certain prescribed set of its elements to zero, in order to achieve a desired network topology. In many cases, the reduction process using these successive 2-dimensional rotations does not converge. Combinations of the two approaches have also been used [4], where the classical synthesis is first applied, and then 2-dimensional rotations are used to transform the coupling matrix to a desired topology.

In many practical cases it is desirable to define the topology of the filter to conform to certain mechanical and packaging constraints. Also, realization of asymmetric structures as well as asymmetric filter responses using the minimum number of resonators is desired. In these cases it is often not possible to use the above synthesis techniques to achieve the desired designs due to failure of convergence. Direct network optimization has been used with commercial software packages, where the coupling matrix elements are the optimization variables and the difference between the transfer function frequency response of the filter and the frequency specification mask is the basis for the objective error function. However this approach is usually inefficient and often results in non-optimum (local minimum) solutions.

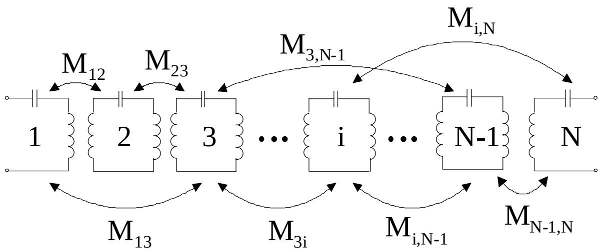

Figure 1: General multiple coupled resonator filter

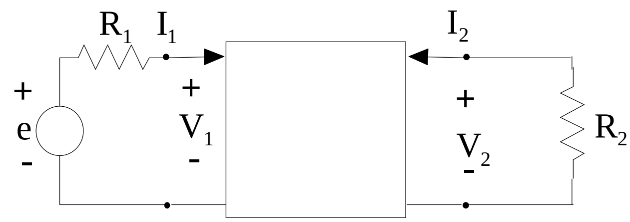

Fig. 2: Filter as a lossless two port network.

This paper presents a different synthesis approach based on using optimization to find the coupling matrix of a prescribed topology. The error function is based on evaluating the reflection coefficient and transmission coefficient at zeros and poles of the desired characteristic function from the trial coupling matrix. The coupling matrices resulting from minimization of the error functions are realizations that approximate the polynomials of the numerators and denominators of the desired filter characteristic function.

PROBLEM STATEMENT AND CIRCUIT ANALYSIS

Consider the equivalent circuit of the multiple coupled resonator filters shown in Fig. 1. The circuit consists of N series resonators with frequency independent couplings M_{ij} between cavities i and j, driven from a source of internal resistance R_1 and terminated in a resistive load R_2. Cavity i has resonant frequency $f_{oi} = f_o + \Delta_i$ where f_o is the center frequency of the filter. This circuit model is valid for narrow band filters (with bandwidth < 20%). The topology of the filter is defined by a topology matrix T, which is an N x N matrix with $T_{ij} = 1$ if M_{ij} is non-zero, and $T_{ij} = 0$ if M_{ij} is zero. Given an insertion loss ratio:

$$S_{21} = \frac{1}{1+\varepsilon^2\Phi^2(\lambda)}$$

where $\lambda = (f_o/BW)[f/f_o - f_o/f]$ *is* the normalized frequency variable, ε is a scale factor related to the pass band ripple, and

$$\Phi(\lambda) = \frac{\prod_{i=1}^{N}(\lambda - A_i)}{\prod_{j=1}^{m}(\lambda - B_j)}$$

is the characteristic function, the problem is to find the coupling matrix M with a given topology T that realizes the desired insertion loss ratio.

The filter network is a lossless two port network, driven by a source of internal resistance R_1 and terminated in a load R_2, as shown in Fig. 2. Analysis of this network leads to the following expressions for the insertion loss ratio S_{21} and the reflection coefficient S_{11}:

$$S_{21} = \frac{-2j\sqrt{R_1R_2}\,P_{12}}{D + P_oR_1R_2 - j\left(P_{11}R_1 + P_{22}R_2\right)}$$

$$S_{11} = \frac{D - P_oR_1R_2 + j\left(P_{11}R_1 - P_{22}R_2\right)}{D + P_oR_1R_2 - j\left(P_{11}R_1 + P_{22}R_2\right)}$$

where P_{11}, P_{12}, P_{22}, P_o, and D are polynomials in λ related to the coupling matrix M by:

$$[\lambda I - M]^{-1}{}_{11} = \frac{P_{11}}{D} \qquad [\lambda I - M]^{-1}{}_{22} = \frac{P_{12}}{D}$$

$$P_0D = P_{12}^2 - P_{11}P_{22} \qquad [\lambda I - M]^{-1}{}_{12} = \frac{P_{12}}{D}$$

All the required polynomials can be readily obtained using the Souriau-Frame algorithm [9] for the inversion of the matrix *[λI-M]*.

SYNTHESIS BY OPTIMIZATION AND NUMERICAL EXAMPLES

The synthesis procedure starts by an initial guess for the coupling matrix. Unlike many other optimization methods, the choice of the initial guess appears to have no measurable effect on the final result or the computation time in most cases, which indicates that the convergence is to a global minimum. A simple initial guess for the coupling matrix is the topology matrix, i.e. M=T. An alternative choice of the starting coupling matrix guess is the Tchebycheff coupling matrix for a filter of the same order. Regardless of the initial guess the optimization always converges to a desirable solution. The error function is defined as follows:

$$Errf = \sum_{i=1}^{N}[S_{11}(A_i)]^2 + \sum_{j=1}^{m}\left[S_{21}(B_j)\right]^2 + [\varepsilon - \hat{\varepsilon}]^2$$

where the functions S_{11}, S_{21}, and $\hat{\varepsilon}$ are evaluated from the current trial matrix M, and ε is the desired value of the scale factor related to the pass band ripple as defined in the insertion loss ratio. A standard gradient unconstrained search minimization algorithm is used to minimize the error function. Convergence of the minimization is very fast and in all cases tested is independent of the initial coupling matrix guess. In contrast, optimization using an error function based on the difference between the mask and the response was slow, often did not converge to any acceptable solution, and in all cases required an initial coupling matrix guess whose response was close to the desired response in order to converge.

A computer program was developed to perform the synthesis by optimization described above. Numerous examples were run to verify and test the program. Some of these examples are summarized in Table 1. In all cases, the initial coupling matrix guess was chosen as the topology matrix. The CPU times indicated in the table is for a 200-MHz Pentium processor PC. Typical filter responses included in Table 1 and their optimized coupling matrices are shown in Fig. 3. Fig. 3(a) is the symmetric response of a 6-pole filter, having one finite transmission zero, on either side of the pass band. Fig. 3(b) is the response of an asymmetric 6-pole filter with one finite transmission zero in the lower stop band and two finite transmission zeros in the upper stop band. Fig. 3(c) is the response of a 5-pole filter with two finite transmission zeros in the lower stop band. In all cases the synthesized coupling matrix reproduced the reflection and transmission zeros at the specified locations, thus demonstrating the validity of the synthesis procedure.

Filter Type (# of Poles, Zeros)	Start Error	Final Error	CPU (sec)	Response
6-P, 2-Z, Symmetric Response	539	2.08×10^{-3}	0.27	Fig. 3(a)
6-P, 3-Z, Asymmetric Response	19690	2.52×10^{-3}	1.75	Fig. 3(b)
5-P, 2-Z, Asymmetric Response	245	5.15×10^{-4}	0.71	Fig. 3(c)

Table 1 Examples of the Synthesis by Optimization results.

CONCLUSIONS

A synthesis procedure for multiple coupled resonator filters using optimization is introduced. The procedure yields a coupling matrix with a given topology that produces the zeros and poles of the specified characteristic function. The procedure is insensitive to the starting values of the coupling matrix, and converges very fast. Typical examples of practical filters are given which show the effectiveness of the procedure.

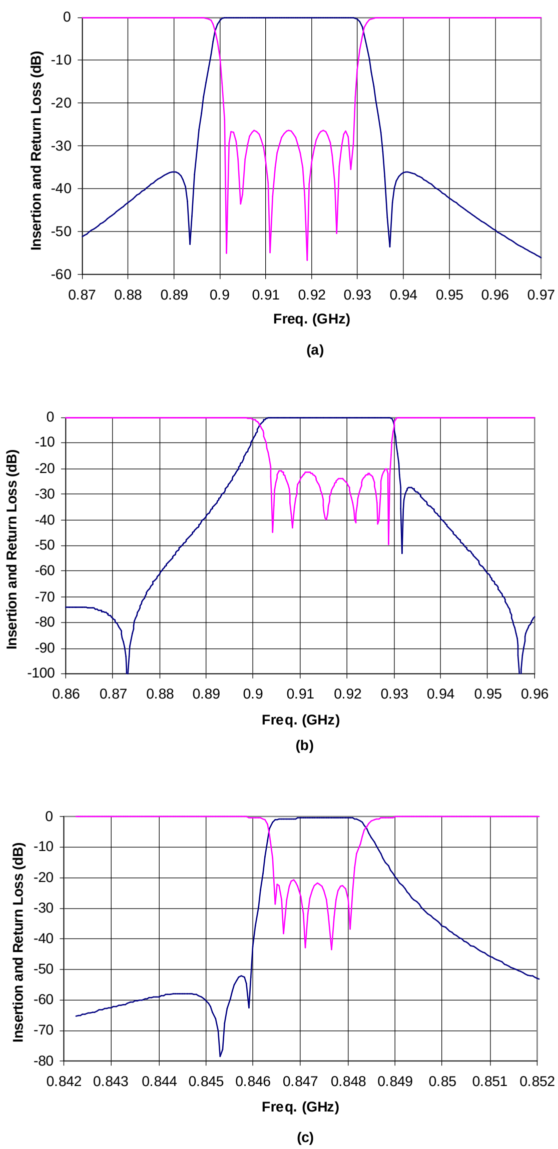

Figure 3: Examples of optimized filter responses: (a) 6-Pole symmetric response 1, (b) 6-Pole asymmetric response, (c) 5-pole asymmetric response.

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